NUMERICAL ANALYSIS
&
ALGORITHMS

Isaac Newton & Joseph Raphson

Newton-Raphson Method

Root-finding via tangent linear approximations and quadratic convergence

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01. CORE FORMULATION

Solving Nonlinear Equations f(x) = 0

The Analytical Challenge

Transcendental and higher-order polynomial equations lack closed-form algebraic solutions. Numerical root-finding yields rapid, arbitrary-precision approximations.

The Newton-Raphson Idea

Replace the non-linear curve locally with its tangent line. The tangent's x-intercept becomes a substantially refined guess for the true root.

02. GEOMETRIC MECHANISM

How Tangent Linearization Drives Convergence

01

Initial Guess (x_0)

Evaluate function point (x_0, f(x_0)) on curve.

02

Construct Tangent

Compute local derivative gradient slope f'(x_0).

03

Next Estimate (x_1)

Find tangent line x-intercept to obtain closer root guess.

Derivation via First-Order Taylor Series

Step 1: Expand f(x) around current point x_n

$$f(x) \approx f(x_n) + f'(x_n)(x - x_n)$$

Step 2: Set root approximation target f $(x_{n+1})=0$

$$0 = f(x_n) + f'(x_n)(x_{n+1} - x_n)$$

Step 3: Isolate algebraic difference term

$$f'(x_n)(x_{n+1} - x_n) = -f(x_n)$$

Final Update Formula:

$$x_{n+1} = x_n - [\, f(x_n) / f'(x_n) \,]$$

04. ALGORITHMIC EXECUTION

Step-by-Step Computational Workflow

1

Choose x_0

Select initial estimate close to true root.

2

Compute Derivatives

Evaluate values for $f(x_n)$ and $f'(x_n)$.

3

Update Guess

Apply $x_{n+1} = x_n - f(x_n)/f'(x_n)$.

4

Check Tolerance

Stop when $|x_{n+1} - x_n| < \varepsilon$.

05. PRACTICAL NUMERICAL WALKTHROUGH

Solving $f(x) = x^2 - 4x - 7 = 0$ (Initial Guess x_0=0.5)

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	5.000000	2.000000	6.000000	5.333333	0.333333
1	5.333333	0.111111	6.666667	5.316667	0.016666
2	5.316667	0.000278	6.633333	5.316625	0.000042
3	5.316625	0.000000	6.633250	5.316625	0.000000

06. CONVERGENCE RATE

Quadratic Convergence Rate: $e_{n+1} \approx C \cdot e_n^2$

2x

Precision Precision Doubling

Number of correct decimal places doubles with each successful iteration step.

$O(e^2)$

Error Magnitude Reduction

Subsequent error proportional to squared prior error, ensuring rapid terminal speed.

07. KEY STRENGTHS

Why Newton-Raphson is Widely Adopted

Rapid Execution

Minimal iterations needed near root targets.

Low Memory Footprint

Requires storing only single state variable.

Multidimensional

Extends cleanly to systems using Jacobian matrices.

08. FAILURE MODES

Critical Limitations & Pathologies

Zero Derivative

$f'(x_n) = 0$ induces division by zero error.

Infinite Cycles

Inflection points cause infinite step oscillations.

Divergence

Poor initial guesses shoot estimate to infinity.

Local Extrema Traps

Near-zero derivative shoots x_{n+1} far away.

09. COMPARATIVE BENCHMARK

Newton-Raphson vs. Alternative Root Finders

Method	Order of Convergence	Derivative Required?	Guaranteed Convergence?
Newton-Raphson	**Quadratic (Order 2)**	**Yes — f'(x) required**	**No (Conditional)**
Bisection Method	Linear (Order 1)	No	**Yes (If bracketed)**
Regula Falsi	Linear (Order 1)	No	**Yes (If bracketed)**
Secant Method	Superlinear (~1.618)	No	No (Conditional)

Thank You….